

Method Study of Parameter Choice for a Circular Proton-Proton Collider ^{*}

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Abstract: In this paper we showed a systematic method of appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift limit started from given design goal and technical limitations. A parameter space has been explored. Based on parameters scan and considerations from RF systems, a set of appropriate parameter designed for a 50Km and a 100Km circular proton-proton collider was proposed.

Key words: circular proton-proton collider, parameter choice, beam-beam tune shift limit

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1 Introduction

With the discovery of Higgs boson on LHC, the world high-energy physics community is investigating the feasibility of a Higgs Factory as a complement to the LHC for studying the Higgs and interested in the frontier of high energy. The CERN people are busy planning the LHC upgrade program, including HL-LHC and HE-LHC. They also plan a more inspiring program called FCC, including FCC-ee and FCC-hh. Both the HE-LHC and the FCC-hh are proton-proton colliders aiming to explore the high energy frontier and expecting to find new physics [1][2][3][4]. Chinese accelerator physicists also plan to design an ambitious machine called CEPC-SPPC (Circular Electron Positron Collider-Super Proton Proton Collider). The CEPC-SPPC program contains two stage. The first stage is an electron-positron collider with center-of-mass energy 240GeV to study Higgs properties carefully. The second stage is a proton-proton collider at center-of-mass energy more than 70TeV [5][6][7]. The SPPC design is just starting. We developed a systematic method of how to make an appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift started from the required luminosity goal, beam energy, physical constraints at IP and some technical limitations.

2 Beam-Beam tune shift limit

In storage ring colliders, due to quantum excitation and synchrotron damping effects, the particles are con-

fined inside a bunch. In e^+e^- colliders, the quantum excitation is very strong and the position for each particle is random and the state of the particles can be regarded as a gas, where the positions of the particles follow statistic laws. Apparently, the synchrotron radiation is the main source of heating. Besides, when two bunches undergo collision at an interaction point (IP), every particle in each bunch will feel the deflected electromagnetic field of the opposite bunch and the particles will suffer from additional heatings. With the increase of the bunch particle population N_e , this kind of heating effect will get stronger. There is a limit condition beyond which the beam emittance will blow up. This emittance blow-up mechanism introduce a limit for beam-beam tune shift which was well discussed in reference [8]:

$$\xi_{y,max} \leq 2845\gamma \sqrt{\frac{r_p}{6\pi R N_{IP}}} = \frac{2845}{2\pi} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} \quad (1)$$

In pp circular colliders, the synchrotron damping effect is very weak. The position for each particle is not like that for electron which is random and the state of the particles cannot be regarded as a gas. Due to the lack of strong synchrotron radiation, the particles inside a bunch are very cold and one can trace each particle without missing it. When the bunches suffer from the strong nonlinear beam-beam forces, some particles located in the outer part of the bunch undergo nonlinear force induced stochastically motions. The number of this heated particles, $N_{p,h}$ can be estimated by $N_{p,h} = f(x)N_p$ [9]. With

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$$f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt \quad (2)$$

Where N_p is the particle number inside a bunch, x is the limit between the cold core and the heated region. On this condition, the limit for beam-beam tune shift can be expressed as [9]:

$$\xi_{y,max} = \frac{2845\gamma}{f(x)} \sqrt{\frac{r_p}{6\pi R N_{IP}}} = \frac{2845}{2\pi f(x)} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} = \frac{\xi_1}{f(x)} \quad (3)$$

$$f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt \quad (4)$$

$$x^2 = \frac{4f(x)}{\pi \xi_{y,max} N_{IP}} = \frac{4f(x)^2}{\pi \xi_1 N_{IP}} \quad (5)$$

Where N_{IP} is the number of interaction point (When there are N_{IP} interaction points, the independent heating effects have to be added in a statistical way), R is the dipole radius, r_p is the classical radius of proton, τ_y is the transverse damping time and T_0 is the revolution time.

3 Machine parameters choice

The design goal of energy of SPPC is about 70-100TeV using the same tunnel with CEPC which is about 50Km. A larger circumference like 100Km for SPPC is also being considered. We want to use the superconducting magnets which is about 20T [10]. We can develop a systematic way to calculate the parameter starting from the maximum beam beam tune shift limit and the design goal. Our design goal is: luminosity L_0 , beam energy E_0 , ring circumference C_0 and IP numbers N_{IP} . Table 1 shows the goals, known quantities and constants.

Table 1. The design goal and known quantities.

Circumference	$C_0 = 54.7Km$
Beam Energy	$E_0 = 35TeV$
IP numbers	$N_{IP} = 2$
Luminosity	$L = 1.0 \times 10^{35} cm^{-2} s^{-1}$
Total straight section length	$L_{SS} = 7595m$
Arc filling factor	$f_1 = 0.79$
Bunch filling factor	$f_2 = 0.80$
Energy gain(15 ~ 20)	$Gain = 16.67$
Total/inelastic cross section	$\sigma_{cross} = 140mbarn$
Light speed	$c = 3 \times 10^8 m/s$

The luminosity for pp collider can be written as [4]:

$$\mathcal{L} = \frac{I_b}{e} \frac{\xi_y}{\beta^*} \frac{\gamma}{r_p} F_{ca} F_h \quad (6)$$

$$\mathcal{L}_0 = \frac{I_b}{e} \frac{\xi_y}{\beta^*} \frac{\gamma}{r_p} \quad (7)$$

Where, F_{ca} is the luminosity reduction factor due to cross angle [11]:

$$F_{ca} = \frac{1}{\sqrt{1 + (\frac{\sigma_z \theta_c}{2\sigma^*})^2}} \quad (8)$$

F_h is the luminosity reduction factor due to hourglass effect [12]:

$$F_h = \frac{\beta^*}{\sqrt{\pi} \sigma_z} \exp(\frac{\beta^{*2}}{2\sigma_z^2}) K_0(\frac{\beta^{*2}}{2\sigma_z^2}) \quad (9)$$

Put $\xi_{y,max}$ into the luminosity formula, we can get:

$$\mathcal{L}_0 = \frac{I_b}{e} \frac{\xi_{y,max}}{\beta^*} \frac{\gamma}{r_p} = \frac{2845}{2\pi r_p e f(x)} \frac{1}{\beta^*} \sqrt{\frac{I_b P_{SR} \gamma}{2E_0 N_{IP}}} \quad (10)$$

And, then the beta function at IP can be written as:

$$\beta^* = \frac{2845}{2\pi r_p e f(x)} \frac{1}{\mathcal{L}_0} \sqrt{\frac{I_b P_{SR} \gamma}{2E_0 N_{IP}}} \quad (11)$$

The RMS IP spot size: ($\sigma^* = \sigma_x = \sigma_y$)

$$\sigma^* = \sqrt{\beta^* \epsilon} = \sqrt{\beta^* \frac{\epsilon_n}{\gamma}} \quad (12)$$

Beta at the 1st parasitic encounter with bunch separation Δt :

$$l_1 = c \times \Delta t \quad (13)$$

$$\beta_1 = \beta^* + \frac{(l_1/2)^2}{\beta^*} \quad (14)$$

RMS spot size at the 1st parasitic encounter:

$$\sigma_1 = \sqrt{\beta_1 \epsilon} = \sqrt{\beta_1 \frac{\epsilon_n}{\gamma}} \quad (15)$$

The full cross angle [4]:

$$\theta_c = \frac{2 \times 6\sigma_1}{l_1/2} = \frac{24\sigma_1}{l_1} \quad (16)$$

We can rewrite F_{ca} as:

$$F_{ca} = \frac{1}{\sqrt{1 + \Phi^2}} \quad (17)$$

$$\begin{aligned} \Phi &= \frac{\sigma_z \theta_c}{2\sigma^*} = \frac{12\sigma_z \sigma_1}{l_1 \sigma^*} = \frac{12\sigma_z \sqrt{\beta_1 \frac{\epsilon_n}{\gamma}}}{l_1 \sqrt{\beta^* \frac{\epsilon_n}{\gamma}}} = \frac{12\sigma_z}{l_1} \sqrt{\frac{\beta_1}{\beta^*}} \\ &= 12 \sqrt{\frac{\sigma_z^2}{(c\Delta t)^2} + \frac{1}{4(\beta^*/\sigma_z)^2}} \end{aligned} \quad (18)$$

Where Φ is Piwinski angle, β^* is beta function at IP, σ_z is bunch length and Δt is the bunch separation.

When the luminosity reduce less than 10% due to the crossing angle effect, we have $F_{ca} \geq 0.9$. From equation(17) we get :

$$\Phi \leq 0.434822(rad) \quad (19)$$

Bunch numbers:

$$n_b = \frac{T_0 f_2}{\Delta t} \quad (20)$$

Bunch population:

$$N_p = \frac{I_b}{n_b f_{rev} e} \quad (21)$$

Combining equation(11)(18)(19)(20)(21), we can get reasonable values of β^* I_b Δt n_b N_p and the ratio β^*/σ_z , where should also consider the instability influence and the constraints from technic.

From the definition of beam beam tune shift [11]:

$$\xi_y = \frac{N_p r_p}{4\pi \epsilon_n} \quad (22)$$

We can get the normalized emittance:

$$\epsilon_n = \frac{N_p r_p}{4\pi \xi_{y,max}} \quad (23)$$

Then we can calculate σ^* β_1 σ_1 θ_c and F_h . Finally, we get the final value of the luminosity:

$$\mathcal{L} = \mathcal{L}_0 F_{ca} F_h \quad (24)$$

We can also calculate the follow parameters easily. Energy loss per turn [13]:

$$U_0 = 0.00778 [MeV] \frac{(E_0 [TeV])^4}{\rho [m]} \quad (25)$$

SR power per ring:

$$P_{SR} = U_0 I_b \quad (26)$$

Critical photon energy $[E_c]$ [13][14]:

$$E_c [KeV] = 1.077 \times 10^{-4} (E_0 [TeV])^2 B [T] \quad (27)$$

Accumulated particles per beam:

$$N_{ACC} = N_p n_b \quad (28)$$

Stored energy per beam:

$$W = N_{ACC} E_0 e = N_p n_b E_0 e \quad (29)$$

ARC SR heat load [15]:

$$SR \text{ heat load} = \frac{P_{SR}}{L_{Dipole}} \quad (30)$$

Transverse damping time $[\tau_x]$ [16]:

$$\tau_x = \frac{2E_0 T_0}{J_x U_0} \quad (31)$$

Longitudinal damping time $[\tau_\epsilon]$ [16]:

$$\tau_\epsilon = \frac{2E_0 T_0}{J_\epsilon U_0} \quad (32)$$

Beam life time due to burn-off [11]:

$$\tau_{burn-off} = \frac{N_p n_b}{\mathcal{L} N_{IP} \sigma_{cross}} = \frac{N_{ACC}}{\mathcal{L} N_{IP} \sigma_{cross}} \quad (33)$$

The time required to reach 1/e of the initial luminosity [11]:

$$\tau_{1/e} = (\sqrt{e} - 1) \times \tau_{burn-off} \quad (34)$$

Other contributions to luminosity decay come from Toucheck scattering and from particle losses due to a slow emittance blow-up. An emittance blow-up can be caused by the scattering of particles on the residual gas, the nonlinear force of the beam-beam interaction, RF noise and IBS scattering effects. The synchrotron radiation damping decreases the bunch dimensions and can partially compensate the beam size blow-up due to the above effects. Assuming that the radiation damping process just cancels the beam blow up due to the beam-beam interactions and RF noise, one can estimate the net luminosity lifetime by [11]:

$$\tau_L = \frac{1}{\frac{1}{\tau_{IBS}} + \frac{2}{\tau_{rest-gas}} + \frac{1}{\tau_{1/e}}} \quad (35)$$

If the run time τ_{run} fulfils equation(36), the integrated luminosity has the maximum value and the run time will be the optimum run time [11].

$$\log\left(\frac{\tau_{turn-around} + \tau_{run}}{\tau_L} + 1\right) = \frac{\tau_{run}}{\tau_L} \quad (36)$$

$$\tau_{optimum} = \tau_{run} \quad (37)$$

Integrating the luminosity over one luminosity run $[fb^{-1}]$:

$$L_{int} = \mathcal{L} \tau_L (1 - e^{-\frac{\tau_{run}}{\tau_L}}) \times \frac{3600}{10^{39}} \quad (38)$$

where τ_{run} is the optimum total length of the luminosity run.

The overall collider efficiency depends on the ratio of the run length and the average turnaround time. So the optimum average integrated luminosity/day $[fb^{-1}]$ is [11]:

$$L_{tot} = \frac{24}{\tau_{run}[h] + \tau_{turn-around}[h]} L_{int} \quad (39)$$

As a summary, we obtain a set of machine parameters with luminosity goal L_0 , beam energy E_0 , ring circumference C_0 and IP numbers N_{IP} .

$$U_0 = 0.00778 [MeV] \frac{(E_0 [TeV])^4}{\rho [m]} \quad (40)$$

$$E_c [KeV] = 1.077 \times 10^{-4} (E_0 [TeV])^2 B [T] \quad (41)$$

$$P_{SR} = U_0 I_b \quad (42)$$

$$\xi_{y,max} = \frac{2845\gamma}{f(x)} \sqrt{\frac{r_p}{6\pi R N_{IP}}} = \frac{2845}{2\pi f(x)} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} = \frac{\xi_1}{f(x)} \quad (43)$$

$$f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt \quad (44)$$

$$x^2 = \frac{4f(x)}{\pi \xi_{y,max} N_{IP}} = \frac{4f(x)^2}{\pi \xi_1 N_{IP}} \quad (45)$$

$$\mathcal{L}_0 = \frac{I_b}{e} \frac{\xi_{y,max}}{\beta^*} \frac{\gamma}{r_p} = \frac{2845}{2\pi r_p e f(x)} \frac{1}{\beta^*} \sqrt{\frac{I_b P_{SR} \gamma}{2E_0 N_{IP}}} \quad (46)$$

$$\beta^* = \frac{2845}{2\pi r_p e f(x)} \frac{1}{\mathcal{L}_0} \sqrt{\frac{I_b P_{SR} \gamma}{2E_0 N_{IP}}} \quad (47)$$

$$\sigma^* = \sqrt{\beta^* \epsilon} = \sqrt{\beta^* \frac{\epsilon_n}{\gamma}} \quad (48)$$

$$l_1 = c \times \Delta t \quad (49)$$

$$\beta_1 = \beta^* + \frac{(l_1/2)^2}{\beta^*} \quad (50)$$

$$\sigma_1 = \sqrt{\beta_1 \epsilon} = \sqrt{\beta_1 \frac{\epsilon_n}{\gamma}} \quad (51)$$

$$\theta_c = \frac{2 \times 6\sigma_1}{l_1/2} = \frac{24\sigma_1}{l_1} \quad (52)$$

$$F_{ca} = \frac{1}{\sqrt{1 + (\frac{\sigma_z \theta_c}{2\sigma^*})^2}} = \frac{1}{\sqrt{1 + \Phi^2}} \quad (53)$$

$$\Phi = \frac{\sigma_z \theta_c}{2\sigma^*} = 12 \sqrt{\frac{\sigma_z^2}{(c\Delta t)^2} + \frac{1}{4(\beta^*/\sigma_z)^2}} \quad (54)$$

$$n_b = \frac{T_0 f_2}{\Delta t} \quad (55)$$

$$N_p = \frac{I_b}{n_b f_{rev} e} \quad (56)$$

$$\epsilon_n = \frac{N_p r_p}{4\pi \xi_{y,max}} \quad (57)$$

$$F_h = \frac{\beta^*}{\sqrt{\pi} \sigma_z} \exp\left(\frac{\beta^{*2}}{2\sigma_z^2}\right) K_0\left(\frac{\beta^{*2}}{2\sigma_z^2}\right) \quad (58)$$

$$\mathcal{L} = \mathcal{L}_0 F_{ca} F_h \quad (59)$$

$$N_{ACC} = N_p n_b \quad (60)$$

$$W = N_{ACC} E_0 e = N_p n_b E_0 e \quad (61)$$

$$\text{SR heat load} = \frac{P_{SR}}{L_{Dipole}} \quad (62)$$

$$\tau_x = \frac{2E_0 T_0}{J_x U_0} \quad (63)$$

$$\tau_\epsilon = \frac{2E_0 T_0}{J_\epsilon U_0} \quad (64)$$

$$\tau_{burn-off} = \frac{N_{ACC}}{\mathcal{L} N_{IP} \sigma_{cross}} \quad (65)$$

$$\tau_{1/e} = (\sqrt{e} - 1) \times \tau_{burn-off} \quad (66)$$

$$\tau_L = \frac{1}{\frac{1}{\tau_{IBS}} + \frac{2}{\tau_{rest-gas}} + \frac{1}{\tau_{1/e}}} \quad (67)$$

$$\log\left(\frac{\tau_{turn-around} + \tau_{run}}{\tau_L} + 1\right) = \frac{\tau_{run}}{\tau_L} \quad (68)$$

$$\tau_{optimum} = \tau_{run} \quad (69)$$

$$L_{int} = \mathcal{L} \tau_L (1 - e^{-\frac{\tau_{run}}{\tau_L}}) \times \frac{3600}{10^{39}} \quad (70)$$

$$L_{tot} = \frac{24}{\tau_{run}[h] + \tau_{turn-around}[h]} L_{int} \quad (71)$$

4 Compare the LHC parameter list with the parameter obtained by our method

To check our method, we use it to chose and calculate the LHC parameters and compare them with the LHC parameter list[15]. The second column in Table 2 is the parameter obtained using our systematical method, which is reasonable and nearly with the parameters in LHC parameter list. This indicates that our method is reasonable and more powerful. We can use this method to design and choose parameters for any proton proton circular colliders.

Table 2. Compare the LHC parameter list with the parameter obtained by our method.

	LHC-list	LHC-new	
	Value		Unit
Main parameters and geometrical aspects			
Beam energy[E_0]	7	7	TeV
Circumference[C_0]	26.7	26.7	km
Lorentz gamma[γ]	7463	7463	
Dipole field[B]	8.33	8.26	T
Dipole curvature radius[ρ]	2801	2826	m
Bunch filling factor[f2]	0.78	0.80	
Arc filling factor[f1]	0.79	0.79	
Total dipole magnet length[L_{Dipole}]	17599	17756	m
Arc length[L_{ARC}]	22476	22476	m
Total straight section length[L_{ss}]	4224	4224	m
Energy gain factor in collider rings	15.6	15.6	
Injection energy [E $_{inj}$]	0.45	0.45	TeV
Number of IPs[N_{IP}]	4	2	
Physics performance and beam parameters			
Peak luminosity per IP[L]	1.0E+34	1.0E+34	/cm ² s
Optimum run time	15.2	10.46	hour
Optimum average integrated luminosity/day	0.47	0.42	fb ⁻¹
Assumed turnaround time	6	5	hour
Overall operation cycle	21.2	16.0	hour
Beam life time due to burn-off[τ]	45	40.65	hour
Total / inelastic cross section[σ]	111/85	111/85	mbarn
Beam parameters			
Beta function at collision[β^*]	0.55	0.56	m
Max beam-beam tune shift perIP[ξy]	0.0033	0.0032	
Number of IPs contributing to ΔQ	3	2	
Max total beam-beam tune shift	0.01	0.0064	
Circulating beam current[I_b]	0.584	0.589	A
Bunch separation[Δt]	25 5	25 5	ns
Number of bunches[n_b]	2808	2848	
Bunch population[Np]	1.15	1.15	10 ¹¹

Normalized RMS transverse emittance[ϵ]	3.75	4.39	μm
RMS IP spot size[σ^*]	16.7	16.09	μm
Beta at the 1st parasitic encounter[β_1]	26.12	32.37	m
RMS spot size at the 1st parasitic encounter[σ_1]	114.6	138	μm
RMS bunch length[σ_z]	75.5	75.7	mm
Accumulated particles per beam	0.32	0.33	10 ¹⁵
Full crossing angle[θ_c]	285	441.16	μrad
Reduction factor according to cross angle[Fca]	0.8391	0.7788	
Reduction factor according to hour glass effect[Fh]	0.9954	0.9956	
Other beam and machine parameters			
Energy loss per turn[U ₀]	0.0067	0.0066	MeV
Critical photon energy[Ec]	0.044	0.044	KeV
SR power per ring[P ₀]	0.0036	0.0039	MW
Stored energy per beam[W]	0.362	0.367	GJ
RF voltage[V _{rf}]	16	16	MV
RF Frequency[f_{rf}]	400.8	400.8	MHz
Revolution frequency[f_{rev}]	11.236	11.236	kHz
Harmonic number	35671	35671	
rms energy spread[δ_e]	1.129	1.124	10 ⁻⁴
Momentum compaction factor [α _p]	3.225	3.26	10 ⁻⁴
Synchrotron tune[ν _s]	1.904	2.057	10 ⁻³
Synchrotron Frequency[fsyn]	21.4	23.12	Hz
Bucket area	8.7	9.4	eVs
Bucket half height(ΔE/E)	0.36	0.35	10 ⁻³
Arc SR heat load per aperture	0.206	0.22	W/m
Damping partition number [Jx]	1	1	
Damping partition number [Jy]	1	1	
Damping partition number [Jε]	2	2	
Transverse damping time [τx]	25.8	26.18	hour
Longitudinal damping time [τε]	12.9	13.09	hour

5 Parameter choice for SPPC

5.1 Parameter scan

Using the method above, we scan the goal luminosity \mathcal{L}_0 with different bending radius ρ , IP numbers N_{IP} and different ratio of β^*/σ_z . Table 3 shows the input parameters. We get some meaningful results which are shown From Fig.1 to Fig.8.

Table 3. Input parameters for machine design.

Energy E_0	Circumference C_0	Goal luminosity \mathcal{L}_0
35.0TeV	54.7Km	$(1 \sim 4) \times 10^{35} \text{cm}^{-2} \text{s}^{-1}$
IP numbers N_{IP}	Bending radius ρ	ratio of β^*/σ_z
2 ~ 4	5.9 ~ 6.5Km	10 ~ 20

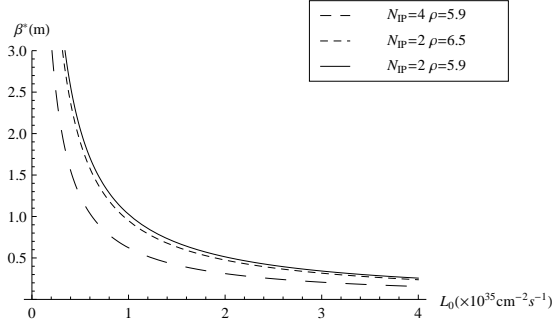


Fig. 1. Vertical beta at IP as the function of goal luminosity.

Fig.1 shows that larger luminosity needs smaller vertical IP beta function. Larger bending radius and more interaction points require smaller β^* at the same goal luminosity.

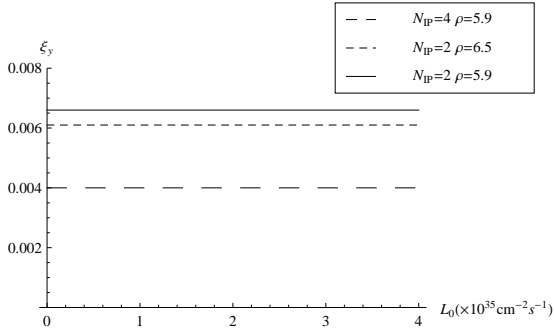


Fig. 2. Vertical beam beam tune shift as the function of peak luminosity.

Fig.2 shows smaller bending radius and less interaction points give larger vertical beam-beam tune shift.

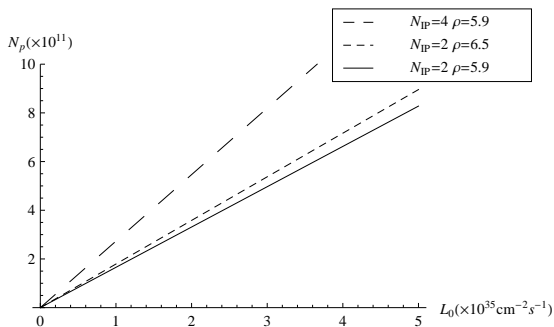


Fig. 3. Bunch population as the function of peak luminosity.

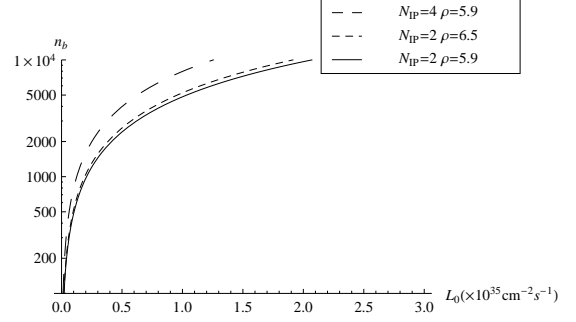


Fig. 4. Bunch number as the function of peak luminosity.

Fig.3 and Fig.4 show that larger luminosity needs larger bunch population or larger bunch number. Larger bending radius and more interaction points indicate larger bunch population or larger bunch number at the same goal luminosity.

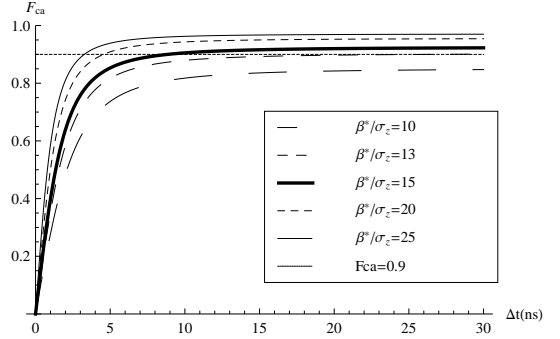


Fig. 5. F_{ca} as the function of Δt .

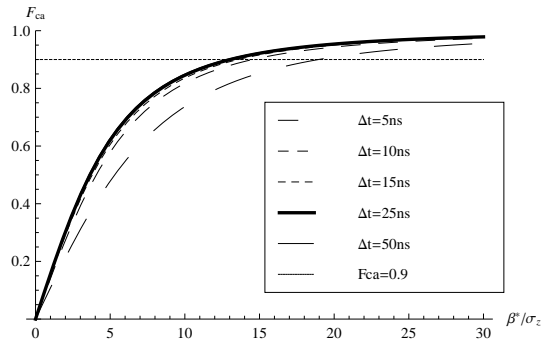


Fig. 6. F_{ca} as the function of the ratio of β^* and σ_z .

Fig.5 and Fig.6 tell us that the reduction factor according to cross angle has relationship with bunch separation(Δt) and the ratio of IP beta and RMS bunch length(β^*/σ_z). The maximum value of this factor is 1, and larger β^*/σ_z makes this value nearer to 1. If we want this effect reduce the luminosity less than 10%, we should have $F_{ca} \geq 0.9$. The dashed line in Fig.5 and Fig.6 is the value equal to 0.9, and we can easily get the important information from the figures. We should choose a larger β^*/σ_z , which about 15 is much reasonable and now the bunch separation is $25ns$. If we want to choose a smaller bunch separation like $5ns$, the ratio of β^* and σ_z should be more than 20. We should consider both of them and choose the eclectic values. Fig.7 shows the 3D diagram of the relationship of F_{ca} , Δt and β^*/σ_z .

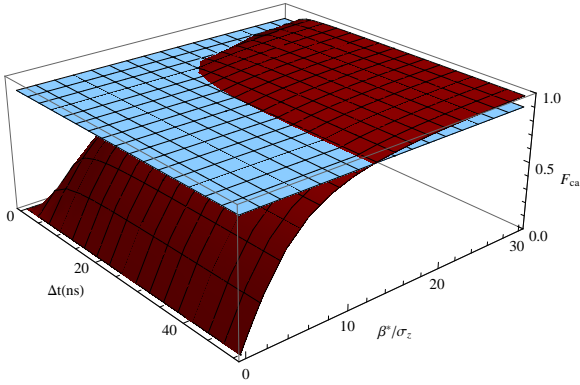


Fig. 7. The 3D diagram of the relationship of F_{ca} , Δt and β^*/σ_z .

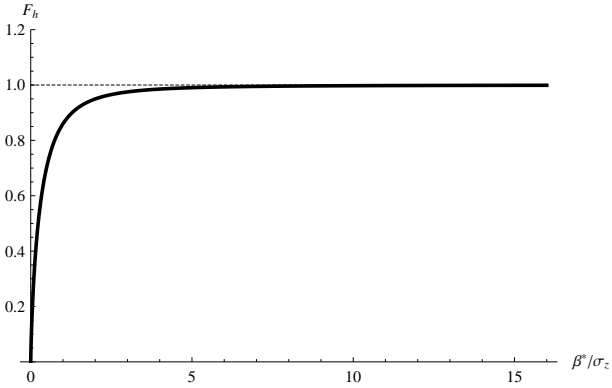


Fig. 8. F_h as the function of the ratio of β^* and σ_z .

Fig.8 shows the reduction factor according to hourglass effect as the function of the ratio of IP β function and RMS bunch length. A large ratio makes larger F_h value. To reduce the reduction of luminosity according to hourglass effect, we should also choose a reasonable larger ratio of β^* and σ_z .

Overall speaking, we should decrease IP numbers and increase bending radius in order to achieve higher luminosity. $N_{IP} = 2$ is the reasonable minimum value for IP

number. Assuming the maximum dipole arc filling factor is 80%, 5.9 km bending radius will be a limit for the 54.7 km ring.

5.2 Constraints form RF system

As long as a set of beam parameters is determined, we need to check the RF system to see if the bunch length can be achieved. Firstly, considering the synchrotron radiation energy loss has to be compensated by the RF cavities, one finds [16]:

$$U_0 = eV_{rf} \sin(\phi_s) \quad (72)$$

where V_{rf} is the total voltage of the RF cavities and ϕ_s is the synchrotron phase. According to eq. (72), one gets

$$\phi_s = \pi - \arcsin\left(\frac{U_0}{eV_{rf}}\right) \quad (73)$$

We can estimate the RF frequency from the "pill-box" model. As the following picture shows. We can find the f_{rf} via the Maxwell's equation and the boundary conditions [16].

$$J_0\left(\frac{\omega}{c}R_0\right) = 0 \quad (74)$$

$$\frac{\omega}{c}R_0 = 2.405 \quad \frac{2\pi f_{rf}}{c}R_0 = 2.405 \quad (75)$$

$$f_{rf} = \frac{2.405c}{2\pi R_0} \quad (76)$$

When the cavity inner radius $R_0 = 30cm$, $f_{RF} = 400MHz$ is a reasonable choose [16].

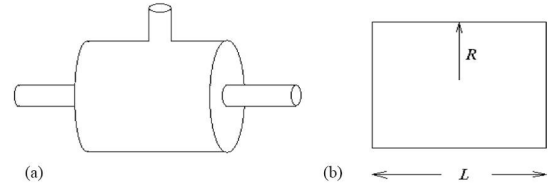


Fig. 9. Pill-box model.

In a storage ring with an isomagnetic guide field (one which has a constant radius ρ in the magnets and is straight elsewhere) the relative energy spread σ_e/E_0 can be expressed as [17]:

$$(\delta_e)^2 = \left(\frac{\sigma_e}{E_0}\right)^2 = \frac{C_q \gamma^2}{J_e \rho_0} \quad (isomag) \quad (77)$$

so,

$$\delta_\epsilon = \gamma \sqrt{\frac{C_q}{J_\epsilon \rho_0}} \quad (78)$$

where $C_q = 1.2817 \times 10^{-12} m$ is a constant.
The nature bunch length is expressed by [17]:

$$\sigma_l = \frac{\alpha_p R \delta_\epsilon}{\nu_s} \quad (79)$$

where, α_p is the momentum compaction factor, R is the average radius of the ring. ν_s is the longitudinal oscillation tune which can be expressed as:

$$\nu_s = \sqrt{-\frac{\eta_p h e V_{rf} \cos \phi_s}{2\pi E_s \beta_s^2}} \quad (80)$$

Where η_p is the phase slippage factor, when $v \approx c$, $\beta \approx 1$, $\gamma \rightarrow \infty$, $\eta_p = \alpha_p - \frac{1}{\gamma^2} \approx \alpha_p$, and $h = f_{rf}/f_{rev} = f_{rf}T_0$, we can rewrite ν_s as follow:

$$\nu_s = \sqrt{-\frac{\alpha_p f_{rf} T_0 e V_{rf} \cos \phi_s}{2\pi E_0}} \quad (81)$$

And then the nature bunch length can be expressed as [17][18]:

$$\sigma_l = \sqrt{-\frac{2\pi E_0 \alpha_p}{f_{rf} T_0 e V_{rf} \cos \phi_s}} R \delta_\epsilon \quad (82)$$

The energy acceptance can be expressed as [17][18]:

$$\eta_{acceptance} = \sqrt{\frac{2U_0}{\pi \alpha_p f_{rf} T_0 E_0} [\sqrt{q^2 - 1} - \arccos(\frac{1}{q})]} \quad (83)$$

where, $q = eV_{rf}/U_0$. Combining the eqs.(82) and eqs.(83), we can get the RF frequency f_{rf} and the momentum compaction α_p for given RF voltage V_{rf} and energy acceptance η .

The synchrotron frequency [18]:

$$f_{syn} = \frac{\nu_s}{T_0} = \nu_s f_{rev} \quad (84)$$

Bucket area [18]:

$$\text{bucket area} = \frac{16\nu_s}{h|\eta_p|\sqrt{|\cos \phi_s|}} \alpha(\phi_s) \quad (85)$$

where the dimensionless function $\alpha(\phi_s)$ is the bucket area normalized to the case when $\phi_s = 0$. For the case $\eta_p < 0$, we have

$$\alpha(\phi_s) = \frac{1}{4\sqrt{2}} \int_{\phi_2}^{\pi - \phi_s} [\cos \phi + \cos \phi_s - (\pi - \pi_s) \sin \phi_s]^{1/2} d\phi \quad (86)$$

when $\phi_s = 0$, $\alpha(\phi_s) = 1$, then the bucket area is $\frac{16\nu_s}{h|\eta_p|}$.

The bucket half height [18]:

$$\text{bucket half height} = \sqrt{\frac{2eV_0 |\cos \phi_s - \frac{\pi - 2\phi_s}{2} \sin \phi_s|}{\pi \beta_s^2 E_s h |\eta_p|}} \quad (87)$$

when $\phi_s = 0$, we have bucket half height $\frac{2\nu_s}{h|\eta_p|}$.

5.3 Machine parameter choice for SPPC

Combining the discussions above, we get a set of new design for the 54.7 km SPPC. We also tried to give a set of parameters for larger circumference SPPC, like 78Km or 100Km. Table 4 is the parameter list for SPPC. As a comparison, we put the parameter for LHC HL-LHC HE-LHC and FCC-hh together in Table 4.[4][10] The first plan for SPPC is using the same tunnel with CEPC. The circumference is 54.7Km which is determined by CEPC. We choose the dipole field as 20T and get center-of-mass energy 70TeV. If we want to explore the higher energy, we should make the circumference larger. When we want to explore center-of-mass energy 100TeV and keep the dipole field 20T, the circumference should be 78Km at least. At this condition, there is hardly space to upgrade. So a 100Km SPPC is much better because the dipole field is only 14.7T at this condition. If we make the dipole field 20T too, we can get the center-of-mass energy as high as 136TeV.

Table 4. Parameter lists for LHC HL-LHC HE-LHC FCC-hh and SPPC.

	LHC	HL-LHC	HE-LHC	FCC-hh	SPPC-Pre-CDR	SPPC-54.7Km	SPPC-100Km	SPPC-100Km	SPPC-78Km	
	Value									Unit
Main parameters and geometrical aspects										
Beam energy[E_0]	7	7	16.5	50	35.6	35.0	50.0	68.0	50.0	TeV
Circumference[C_0]	26.7	26.7	26.7	100(83)	54.7	54.7	100	100	78	km
Lorentz gamma[γ]	7463	7463	14392	53305	37942	37313	53305	72495	53305	
Dipole field[B]	8.33	8.33	20	16(20)	20	19.69	14.73	20.03	19.49	T
Dipole curvature radius[ρ]	2801	2801	2250	10416 (8333.3)	5928	5922.6	11315.9	11315.9	8549.8	m

Bunch filling factor[f_2]	0.78	0.78	0.63	0.79	0.8	0.8	0.8	0.8	0.8	
Arc filling factor[f_1]	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	
Total dipole magnet length [L_{Dipole}]	17599	17599	14062	65412 (52333)	37246	37213	71100	71100	53720	m
Arc length[L_{ARC}]	22476	22476	22476	83200 (66200)	47146	47105	90000	90000	68000	m
Total straight section length[L_{ss}]	4224	4224	4224	16800	7554	7595	10000	10000	10000	m
Energy gain factor in collider rings	15.6	15.6	13.5	15.2	17.0	16.67	17.5	17.5	17.5	
Injection energy [E_{inj}]	0.45	0.45	>1.0	3.3	2.1	2.1	2.9	3.9	2.9	TeV
Number of IPs[N_{IP}]	4	2	2	2	2	2	2	2	2	
Physics performance and beam parameters										
Peak luminosity per IP[L]	1.0E+34	5.0E+34	5.0E+34	5.0E+34	1.2E+35	1.2E+35	1.52E+35	1.02E+36	1.52E+35	$\text{cm}^{-2}\text{s}^{-1}$
Optimum run time	15.2	10.2	5.8	12.1(10.7)	5.87	5.87	6.69	2.47	5.91	hour
Optimum average integrated lu- minosity/day	0.47	2.8	1.4	2.2(2.1)	3.36	3.36	4.84	12.97	4.28	fb^{-1}
Assumed turnaround time	6			5	5	5	5	5	5	hour
Overall operation cycle	21.2			17.4(16.3)	11.5	11.5	12.5	8.0	12.0	hour
Beam life time due to burn-off[τ]	45	15.4	5.7	19.1(15.9)	9.65	9.65	12.74	2.07	9.78	hour
Total / inelastic cross section[σ]	111/85	111/85	129/93	153/108	140	140	155	160	155	mbarn
Beam parameters										
Beta function at collision[β^*]	0.55	0.15 (min)	0.35	1.1	0.75	0.85	0.97	0.24	1.06	m
Max beam-beam tune shift perIP[ξy]	0.0033	0.0075	0.005	0.005	0.006	0.0065	0.0067	0.008	0.0073	
Number of IPs contributing to ΔQ	3	2	2	2	2	2	2	2	2	
Max total beam-beam tune shift	0.01	0.015	0.01	0.01	0.012	0.013	0.0134	0.016	0.0146	
Circulating beam current[I_b]	0.584	1.12	0.478	0.5	1.0	1.024	1.024	1.024	1.024	A
Bunch separation[Δt]	25 5	25 5	25 5	25 5	25	25	25	25	25	ns
Number of bunches[n_b]	2808	2808	2808	10600 (8900) 53000 (44500)	5835	5835	10667	10667	8320	
Bunch population[N_p]	1.15	2.2	1	1.0 0.2	2.0	2.0	2.0	2.0	2.0	10^{11}
Normalized RMS transverse emittance[ϵ]	3.75	2.5	1.38	2.2 0.44	4.10	3.72	3.65	3.05	3.36	μm
RMS IP spot size[σ^*]	16.7	7.1	5.2	6.8	9.0	8.85	7.85	3.04	7.86	μm
Beta at the 1st parasitic encounter[β_1]	26.12	93.9	40.53	13.88	19.5	18.70	16.51	64.1	15.36	m
RMS spot size at the 1st para- sitic encounter[σ_1]	114.6	177.4	62.3	23.9	45.9	43.2	33.6	51.9	31.14	μm
RMS bunch length[σ_z]	75.5	75.5	75.5	80(75.5)	75.5	56.5	65	15.8	70.6	mm
Accumulated particles per beam	0.32	0.62	0.28	1.06(0.89) 5.3(4.45)	1.2	1.17	2.13	2.13	1.66	10^{15}
Full crossing angle[θ_c]	285	590	185	74	73	138	108	166	99	μrad
Reduction factor according to cross angle[Fca]	0.8391	0.314	0.608	0.910	0.8514	0.9257	0.9248	0.9283	0.9248	
Reduction factor according to hour glass effect[Fh]	0.9954	0.9491	0.9889	0.9987	0.9975	0.9989	0.9989	0.9989	0.9989	
Other beam and machine parameters										
Energy loss per turn[U_0]	0.0067	0.0067	0.201	4.6(5.86)	2.10	1.97	4.30	14.7	5.69	MeV
Critical photon energy[Ec]	0.044	0.044	0.575	4.3(5.5)	2.73	2.60	3.97	9.96	5.25	KeV
SR power per ring[P_0]	0.0036	0.0073	0.0962	2.4(2.9)	2.1	2.0	4.4	15.1	5.82	MW
Stored energy per beam[W]	0.362	0.694	0.701	8.4(7.0)	6.6	6.53	17.1	23.21	13.31	GJ

RF voltage[V_{rf}]	16	16	16	16	16	16	16	16	16	MV
RF Frequency[f_{rf}]	400.8	400.8	400.8	400.8	400.8	400.8	400.8	400.8	400.8	MHz
Revolution frequency[f_{rev}]	11.236	11.236	11.236	3.00	5.48	5.48	3.00	3.00	3.84	kHz
Harmonic number	35671	35671	35671	133600	73079	73079	133600	133600	104208	
rms energy spread[δ_e]	1.129	1.13	2.97	4.17	3.9	3.88	4.01	5.46	4.6	10^{-4}
Momentum compaction factor [α_p]	3.225×10^{-4}	3.92×10^{-4}	2.23×10^{-5}	1.48×10^{-6}	3.39×10^{-6}	1.48×10^{-6}	6.79×10^{-7}	6.56×10^{-9}	7.54×10^{-7}	
Synchrotron tune[ν_s]	1.904	2.26	0.35	0.098	0.133	0.088	0.067	0.0036	0.061	10^{-3}
Synchrotron Frequency[f_{syn}]	21.4	25.33	3.93	0.29	0.73	0.48	0.20	0.011	0.24	Hz
Bucket area	8.7	8.6	23.51	27.01	28.63	43.74	39.94	348.55	42.94	eVs
Bucket half height($\Delta E/E$)	0.36	0.32	0.87	0.78	0.96	1.48	1.17	1.94	1.14	10^{-3}
Arc SR heat load per aperture	0.206	0.33	4.35	28.4(44.3)	57.8	54.24	61.9	211.72	108.41	W/m
Damping partition number [Jx]	1	1	1	1	1	1	1	1	1	
Damping partition number [Jy]	1	1	1	1	1	1	1	1	1	
Damping partition number [Jz]	2	2	2	2	2	2	2	2	2	
Transverse damping time [τ_x]	25.8	25.8	2.0	1.08(0.64)	1.71	1.80	2.15	0.86	1.27	hour
Longitudinal damping time [τ_z]	12.9	12.9	1.0	0.54(0.32)	0.85	0.90	1.08	0.43	0.635	hour

6 Comparing beam-beam tune shift of SPPC with LHC HL-LHC HE-LHC and FCC-hh

In the parameter design of LHC HL-LHC HE-LHC and FCC-hh, the CERN people assume the beam-beam tune shift limit as a constant number [4][11]. But we can find the beam-beam parameter has relationship with several parameters. A method to estimate the maximum

beam-beam tune shift limit was developed in refernce [9]. We compare the calculated numbers with the parameter list chosen numbers and find that these calculated numbers by analytical expression are much reasonable according to the real experimental numbers. We can easily get the ratio of the beam-beam tune shift of the list chosen number and the calculated number. The result was shown in Table 5, we can find that HL-LHC’s choice is much overlarge and the other machines’ choices are more reasonable.

Table 5. Comparing beam-beam tune shift of SPPC with LHC HL-LHC HE-LHC and FCC-hh.

	LHC 7TeV	HL- LHC 7TeV	HE- LHC 16.5TeV	FCC- hh 50TeV	SPPC- Pre- CDR 35.6TeV	SPPC- 54.7Km 35TeV	SPPC- 100Km 50TeV	SPPC- 100Km 68TeV	SPPC- 78Km 50TeV
Number of IPs contributing to ΔQ	3	2	2	2	2	2	2	2	2
Max total beam-beam tune shift	0.01	0.015	0.01	0.01	0.012	0.013	0.0134	0.016	0.0146
Max beam-beam tune shift perIP [ξy] (parameter list)	0.0033	0.0075	0.005	0.005	0.006	0.0065	0.0067	0.008	0.0073
Max beam-beam tune shift perIP [ξy] (calculated)	0.00321	0.00321	0.00499	0.00685	0.00662	0.006559	0.006688	0.00801	0.00731
[ξy](parameter list)/ [ξy](calculated)	1.0287	2.3379	1.002	0.7299	0.9063	0.9910	1.001	0.9986	0.9999

7 Conclusion

In this paper, a systematic method of how to make an appropriate parameter choice for a circular pp collider by using analytical expression of beam-beam tune shift limit started from given luminosity goal, beam energy

and technical limitations was developed. By using this method, we reveal the relations of machine parameters with goal luminosity clearly and hence give a parameter choice in an efficient way. We also show the parameter chose for a 50Km SPPC and larger circumference SPPC, like a 78Km SPPC or a 100Km SPPC.

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